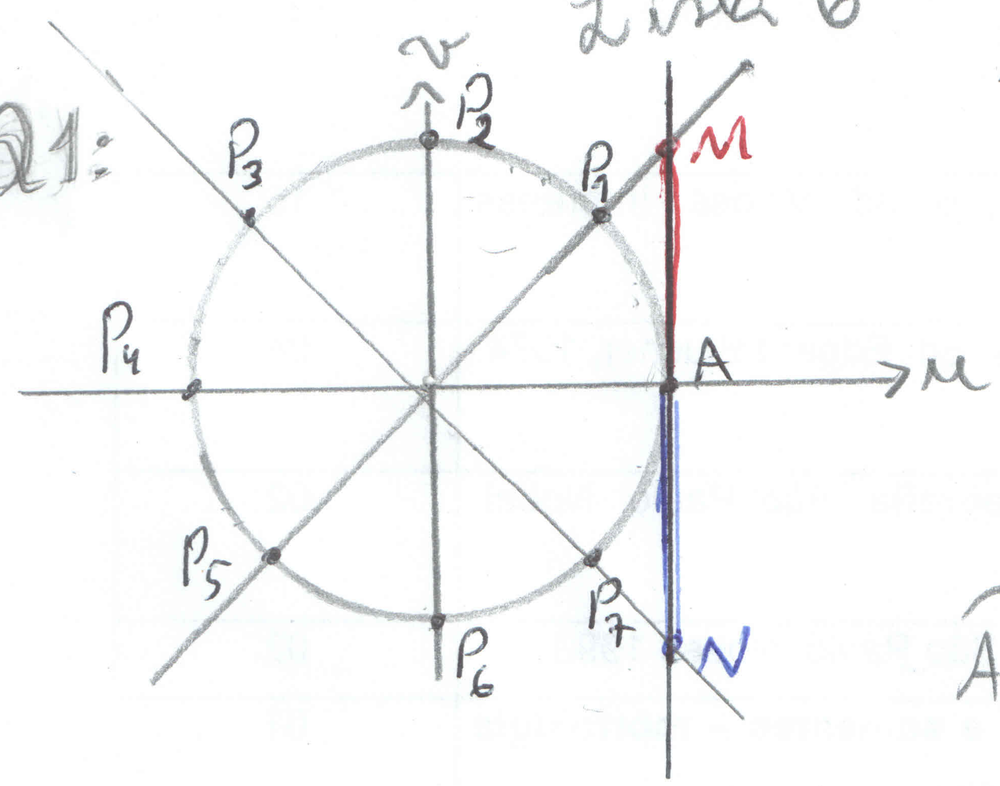
Lista 6

Q1:



$tg\frac{\pi}{4} = \overline{AM} = 1 \qquad \widehat{AP_1} = 45° = \frac{\pi}{4}$

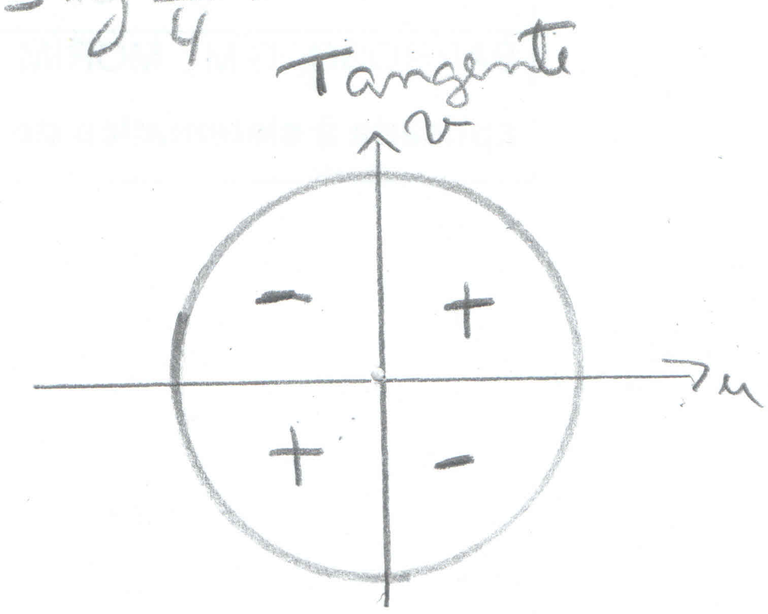
$\widehat{AP_3} = \frac{3\pi}{4} \Rightarrow tg\frac{3\pi}{4} = \overline{AN} = -1$

$\widehat{AP_5} = \frac{5\pi}{4} \Rightarrow tg\frac{\pi}{4} = tg\frac{5\pi}{4} = 1$

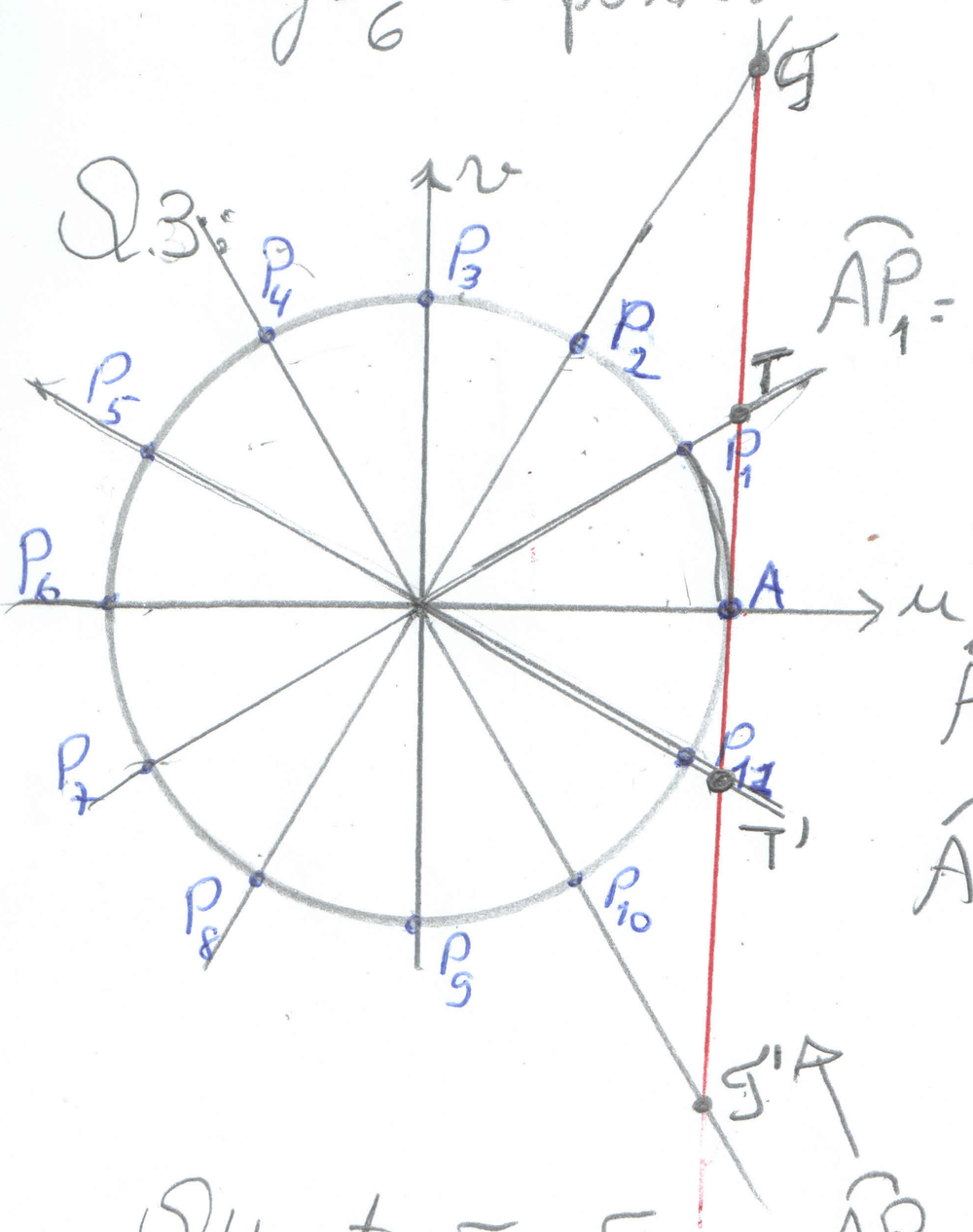
$\widehat{AP_7} = \frac{7\pi}{4} \Rightarrow tg\frac{3\pi}{4} = tg\frac{7\pi}{4} = -1$

Q2: $tg\frac{\pi}{6}$ é positiva $\quad tg\frac{2\pi}{3}$ é negativa

$tg\frac{7\pi}{6}$ é positiva $\quad tg\frac{11\pi}{6}$ é negativa



Q3:



$\widehat{AP_1} = \frac{\pi}{6} \Rightarrow tg\frac{\pi}{6} = \frac{\sqrt{3}}{3} = \overline{AT}$

$\widehat{AP_5} = \frac{5\pi}{6} \Rightarrow tg\frac{5\pi}{6} = -tg\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

$\widehat{AP_7} = \frac{7\pi}{6} \Rightarrow tg\frac{7\pi}{6} = tg\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\widehat{AP_{11}} = \frac{11\pi}{6} \Rightarrow tg\frac{11\pi}{6} = tg\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

Q4: $tg\frac{\pi}{3} = \sqrt{3} \Rightarrow \widehat{AP_2} = \frac{\pi}{3} \Rightarrow \overline{AS} = \sqrt{3}$

$\widehat{AP_4} = \frac{2\pi}{3} \Rightarrow tg\frac{2\pi}{3} = -\sqrt{3}$

$\widehat{AP_8} = \frac{4\pi}{3} \Rightarrow tg\frac{4\pi}{3} = tg\frac{\pi}{3} = \sqrt{3}$

$\widehat{AP_{10}} = \frac{5\pi}{3} \Rightarrow tg\frac{5\pi}{3} = tg\frac{2\pi}{3} = -\sqrt{3}$

Q5:

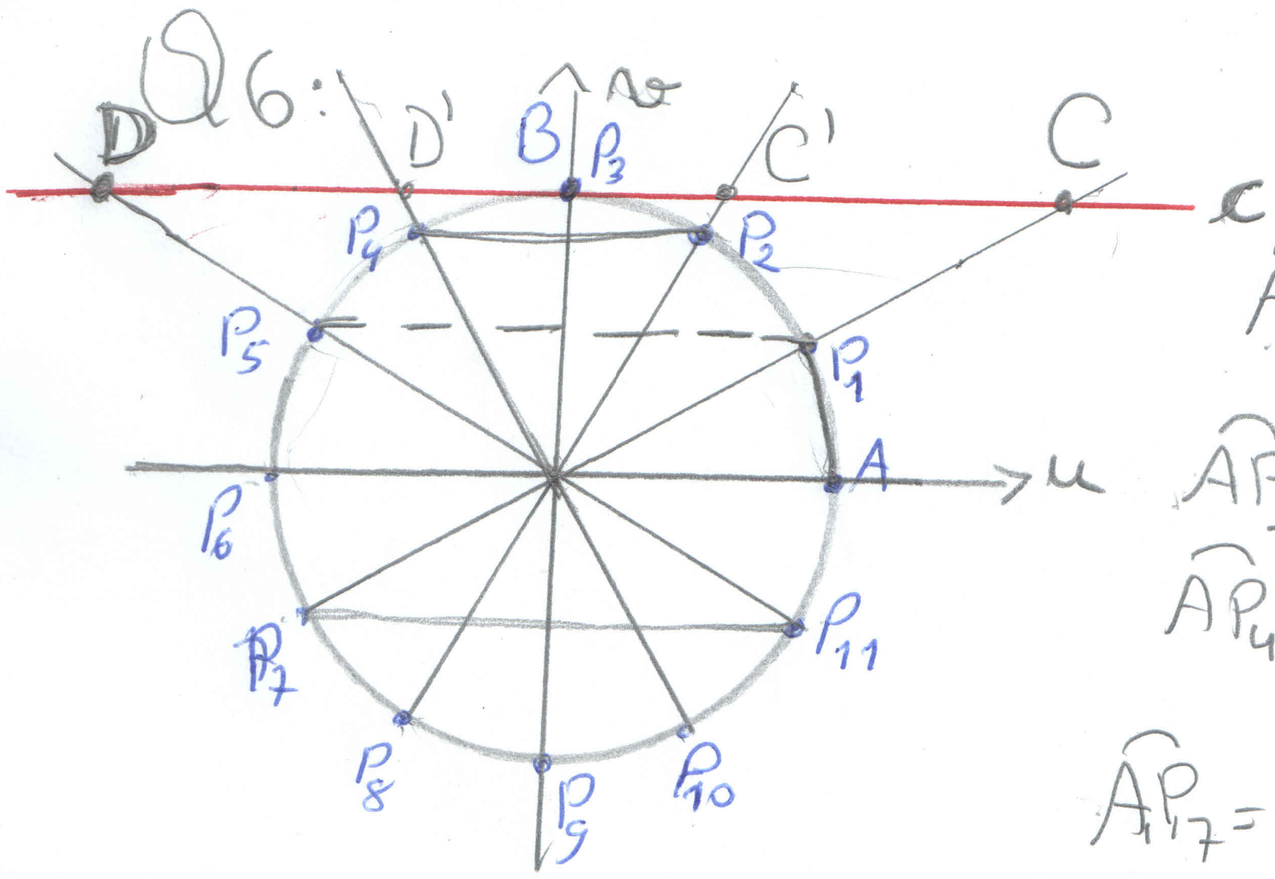
a) $tg\frac{\pi}{3} + tg\frac{\pi}{4} - tg\,2\pi = \sqrt{3} + 1 - 0 = \sqrt{3}+1$

b) $2 \cdot tg\frac{\pi}{6} + \frac{1}{2}\,tg\frac{7\pi}{4} = 2 \cdot \frac{\sqrt{3}}{3} + \frac{1}{2} \cdot (-1) = \frac{2\sqrt{3}}{3} - \frac{1}{2} = \frac{4\sqrt{3}-3}{6}$

c) $-2 \cdot tg\frac{5\pi}{4} + \frac{1}{2}\,tg\,\pi - \frac{1}{3}\,tg\frac{5\pi}{6} = -2 \cdot 1 + \frac{1}{2} \cdot 0 - \frac{1}{3} \cdot \left(-\frac{\sqrt{3}}{3}\right) =$

$-2 + 0 + \frac{\sqrt{3}}{9} = \frac{-18+\sqrt{3}}{9}$

Q6:



$\widehat{AP_1} = \frac{\pi}{6} \Rightarrow cotg\frac{\pi}{6} = \sqrt{3} = \overline{BC}$

$\left.\begin{array}{l}\widehat{AP_2} = \frac{\pi}{3} \\ \widehat{AP_4} = \frac{2\pi}{3}\end{array}\right\} \Rightarrow cotg\frac{2\pi}{3} = -cotg\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

$\widehat{AP_7} = \frac{7\pi}{6} \Rightarrow cotg\frac{7\pi}{6} = cotg\frac{\pi}{6} = \sqrt{3}$

Q7: $cotg\frac{\pi}{6} = \sqrt{3}$

P_1 e P_5 são simétricos em relação ao eixo v, então:

$\widehat{AP_1} = \frac{\pi}{6}$ e $\widehat{AP_5} = \frac{5\pi}{6} \Rightarrow cotg\frac{5\pi}{6} = -cotg\frac{\pi}{6} = -\sqrt{3} = \overline{BD}$

$cotg\frac{7\pi}{6} = cotg\frac{\pi}{6} = \sqrt{3}$

$\widehat{AP_{11}} = \frac{11\pi}{6}$ (P_{11} e P_7 simétricos em relação a $\vec{v}$)

$\Rightarrow cotg\frac{11\pi}{6} = -cotg\frac{7\pi}{6} = -\sqrt{3}$

Q8: $cotg\frac{\pi}{3} = \frac{\sqrt{3}}{3}$ $\left(\widehat{AP_2} = \frac{\pi}{3}\right)$ [P_4 e P_2 simétricos em relação a v]

$\widehat{AP_4} = \frac{2\pi}{3} \Rightarrow cotg\frac{2\pi}{3} = -cotg\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

$\widehat{AP_8} = \frac{4\pi}{3} \Rightarrow cotg\frac{4\pi}{3} = cotg\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

$\widehat{AP_{10}} = \frac{5\pi}{3} \Rightarrow cotg\frac{5\pi}{3} = cotg\frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$

Q9:

a) $\operatorname{sen}\frac{\pi}{3} + \cos\frac{\pi}{4} - \operatorname{tg}\frac{2\pi}{3} + \operatorname{cotg}\frac{7\pi}{6} =$

$\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} - (-\sqrt{3}) + \sqrt{3} = \frac{\sqrt{3}+\sqrt{2}+2\sqrt{3}+2\sqrt{3}}{2} =$

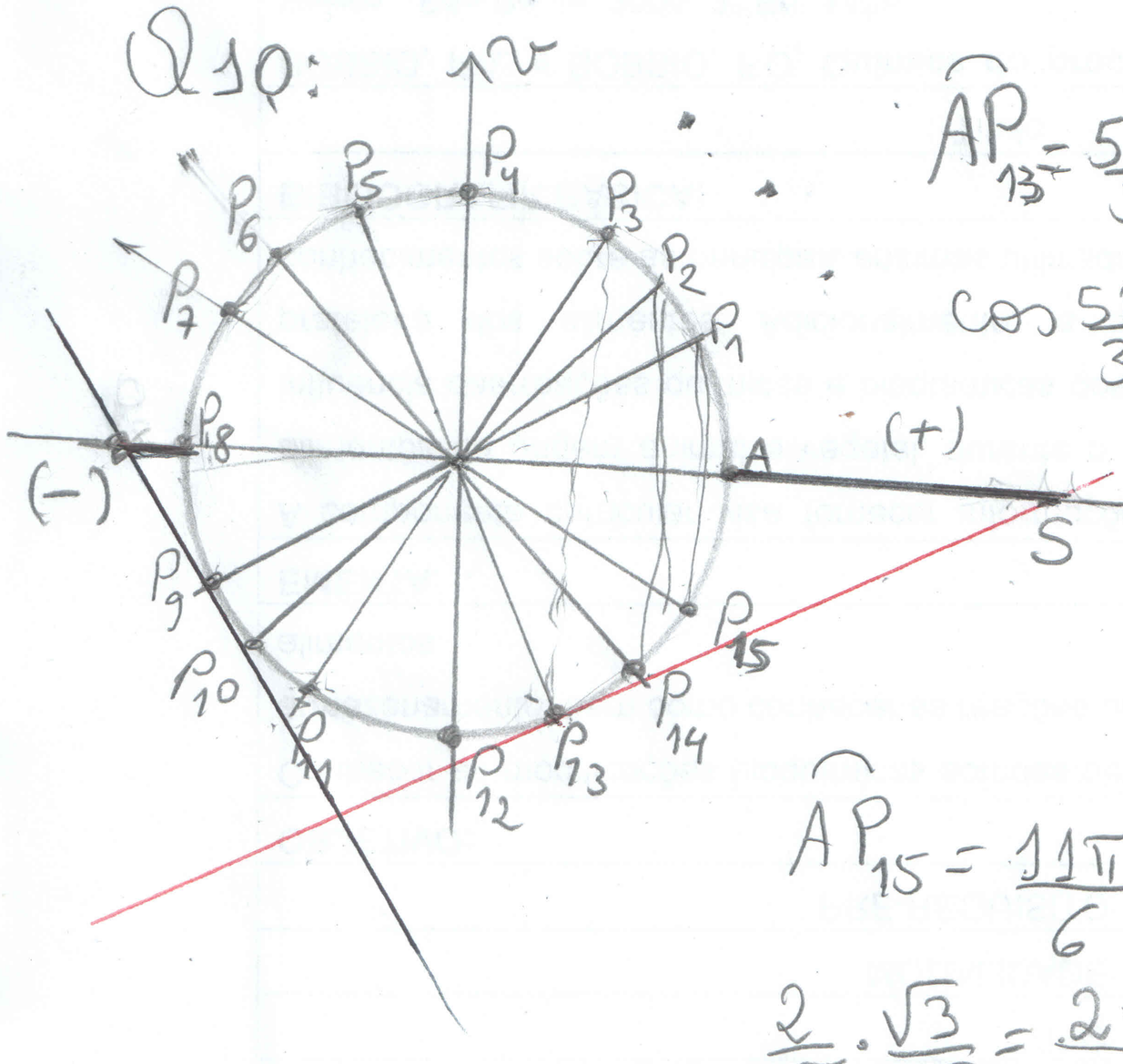
$\frac{5\sqrt{3}+\sqrt{2}}{2}$

b) $\frac{3}{5}\operatorname{cotg}\frac{5\pi}{3} - \frac{6}{7}\operatorname{cotg}\frac{7\pi}{6} - \frac{2}{3}\operatorname{sen}\frac{3\pi}{2} + \frac{4}{5}\cos\frac{5\pi}{4} =$

$\frac{3}{5}\cdot(-\sqrt{3}) - \frac{6}{7}\cdot\sqrt{3} - \frac{\cancel{2}}{3}\cdot\frac{\sqrt{3}}{\cancel{2}} + \frac{\cancel{4}^{2}}{5}\cdot\left(-\frac{\sqrt{2}}{\cancel{2}}\right) = -\frac{3}{5}\sqrt{3} - \frac{6}{7}\sqrt{3} - \frac{\sqrt{3}}{3} - \frac{2}{5}\sqrt{2} =$

$\frac{-63\sqrt{3} - 90\sqrt{3} - 35\sqrt{3} - 42\sqrt{2}}{105} = \frac{-188\sqrt{3} - 42\sqrt{2}}{105}$

Q10:



$\overset{\frown}{AP}_{13} = \frac{5\pi}{3} \Rightarrow \operatorname{sec}\frac{5\pi}{3} = \overline{AS} = \frac{1}{\cos\frac{5\pi}{3}} = \frac{1}{1/2} = 2$

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$ sinal positivo

$\overset{\frown}{AP}_{14} = \frac{7\pi}{4} \Rightarrow \operatorname{sec}\frac{7\pi}{4} = \frac{1}{\cos\frac{7\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} =$

$1\cdot\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ sinal positivo

$\overset{\frown}{AP}_{15} = \frac{11\pi}{6} \Rightarrow \operatorname{sec}\frac{11\pi}{6} = \frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} =$

$\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ sinal positivo

P_9 e P_{15} são simétricos em relação a v, então

$\operatorname{sec}\frac{7\pi}{6} = -\operatorname{sec}\frac{11\pi}{6} = -\frac{2\sqrt{3}}{3}$ sinal negativo

P_{10} e P_{14} são simétricos $\Rightarrow \operatorname{sec}\frac{5\pi}{4} = -\operatorname{sec}\frac{7\pi}{4} = -\sqrt{2}$ sinal negativo

P_7 e P_9 são simétricos em relação a u, então

$\operatorname{sec}\frac{5\pi}{6} = \operatorname{sec}\frac{7\pi}{6} = -\frac{2\sqrt{3}}{3}$ sinal negativo.

Q11: $\sec\frac{\pi}{3}=2$ $\quad \widehat{AP_3}=\frac{\pi}{3}$ $\quad \widehat{AP_5}=\frac{2\pi}{3}$ $\quad \widehat{AP_{11}}=\frac{4\pi}{3}$

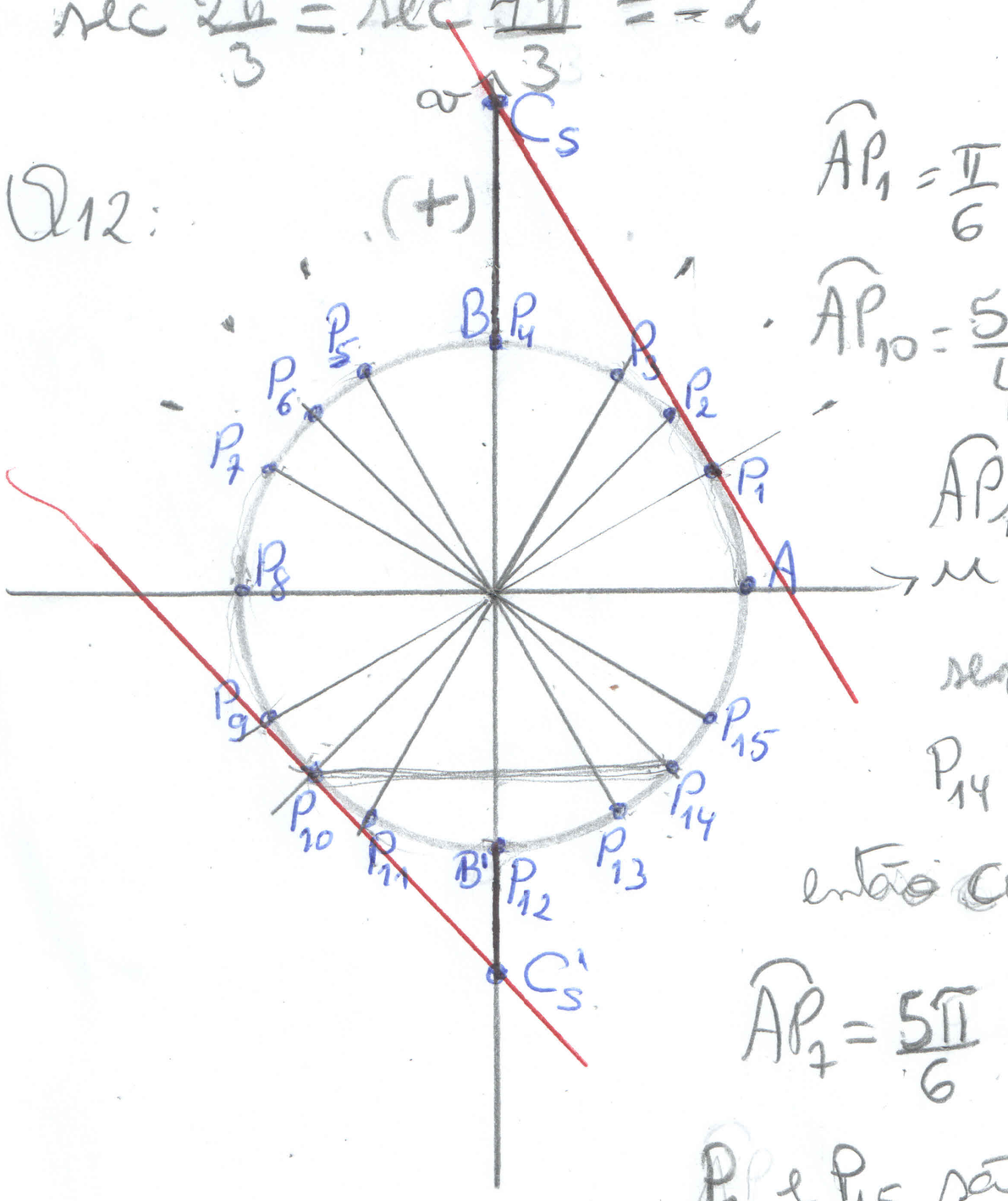
$\widehat{AP_{13}}=\frac{5\pi}{3}$

simetria em relação v	simetria em relação u
P_3 e P_5 $[\widehat{AP_3}$ e $\widehat{AP_5}]$	P_3 e P_{13} $[\widehat{AP_3}$ e $\widehat{AP_{13}}]$
P_{11} e P_{13} $[\widehat{AP_{11}}$ e $\widehat{AP_{13}}]$	P_5 e P_{11} $[\widehat{AP_5}$ e $\widehat{AP_{11}}]$
secantes opostas	secantes iguais

$\sec\frac{4\pi}{3}=-\sec\frac{5\pi}{3}=-2$, pois $\sec\frac{5\pi}{3}=2$

$\sec\frac{2\pi}{3}=\sec\frac{4\pi}{3}=-2$

Q12:



$\widehat{AP_1}=\frac{\pi}{6} \Rightarrow \operatorname{cossec}\frac{\pi}{6}=\overline{BC_s}=\frac{1}{\operatorname{sen}\frac{\pi}{6}}=\frac{1}{1/2}=2$

$\widehat{AP_{10}}=\frac{5\pi}{4} \Rightarrow \operatorname{cossec}\frac{5\pi}{4}=\frac{1}{\operatorname{sen}\frac{5\pi}{4}}=\frac{1}{-\frac{\sqrt{2}}{2}}=-\frac{2}{\sqrt{2}}=-\sqrt{2}$

$\widehat{AP_{14}}=\frac{7\pi}{4}$

$\operatorname{sen}\frac{7\pi}{4}=-\operatorname{sen}\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

P_{14} e P_{10} são simétricos, em relação v,

então $\operatorname{cossec}\frac{7\pi}{4}=\operatorname{cossec}\frac{5\pi}{4}=-\sqrt{2}$

$\widehat{AP_7}=\frac{5\pi}{6}$ $\quad \operatorname{cossec}\frac{5\pi}{6}=\operatorname{cossec}\frac{\pi}{6}=2$

P_7 e P_{15} são simétricos em relação u,

então

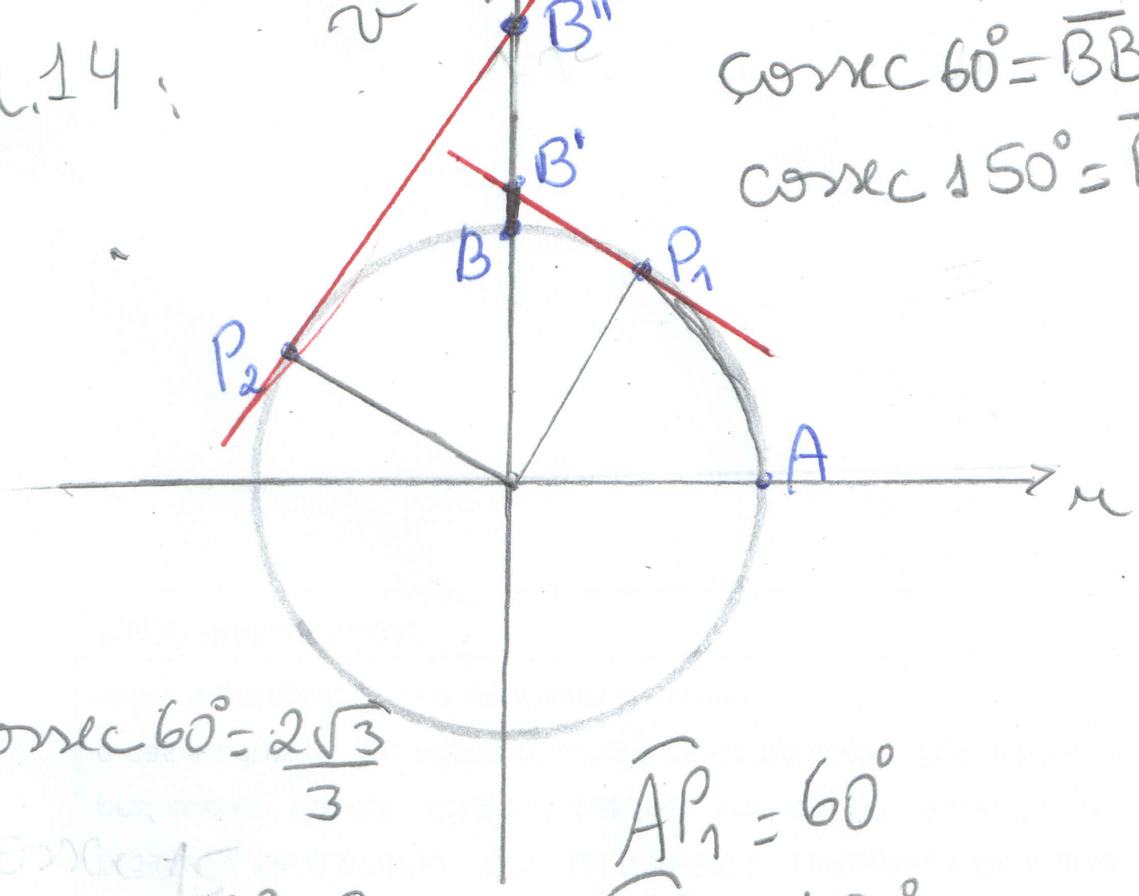
$\widehat{AP_{15}}=\frac{11\pi}{6} \Rightarrow \operatorname{cossec}\frac{11\pi}{6}=-\operatorname{cossec}\frac{\pi}{6}=-2$

Q13: $\operatorname{cossec}\frac{\pi}{3}=\frac{2\sqrt{3}}{3}$ $\quad \operatorname{cossec}\frac{2\pi}{3}=\operatorname{cossec}\frac{\pi}{3}=\frac{2\sqrt{3}}{3}$

$\operatorname{cossec}\frac{4\pi}{3}=-\operatorname{cossec}\frac{2\pi}{3}=-\frac{2\sqrt{3}}{3}$

$\cos\frac{5\pi}{3}=-\operatorname{cossec}\frac{\pi}{3}=-\frac{2\sqrt{3}}{3}$

Q.14:



$\operatorname{cosec} 60° = \overline{BB'}$

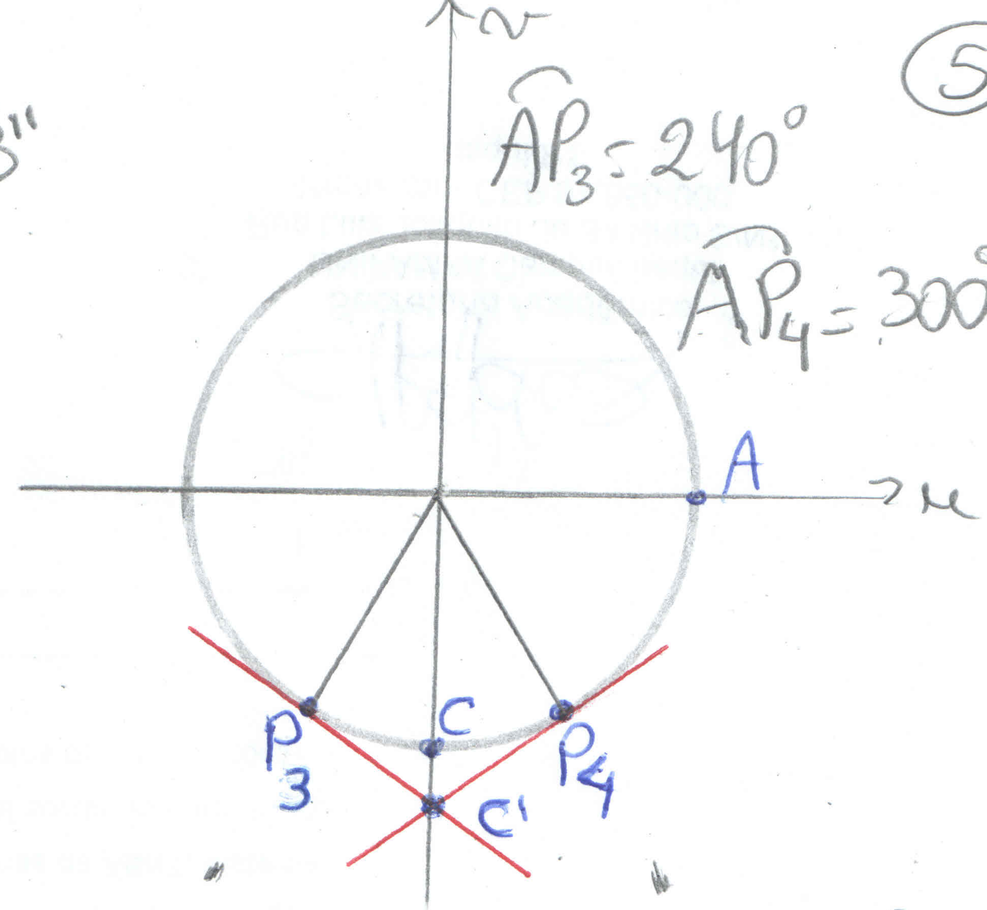
$\operatorname{cosec} 150° = \overline{BB''}$

$\operatorname{cosec} 60° = \frac{2\sqrt{3}}{3}$

$\operatorname{cosec} 150° = 2$

$\widehat{AP_1} = 60°$

$\widehat{AP_2} = 150°$



$\widehat{AP_3} = 240°$

$\widehat{AP_4} = 300°$

$\operatorname{cosec} 240° = \operatorname{cosec} 300° = -\operatorname{cosec} 60° = -\frac{2\sqrt{3}}{3}$

Q15: $\left(\operatorname{cosec}\frac{\pi}{6} + \operatorname{sen}\frac{\pi}{6}\right)\cdot\left(\operatorname{sen}\frac{\pi}{4} - \sec\frac{\pi}{3}\right) =$

$\left(\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{2}\right)\cdot\left(\frac{\sqrt{2}}{2} - 2\right) = \left(\frac{4\sqrt{3}+3\sqrt{3}}{6}\right)\cdot\left(\frac{\sqrt{2}-4}{2}\right) =$

$\frac{7\sqrt{3}}{6}\cdot\frac{(\sqrt{2}-4)}{2} = \frac{7\sqrt{6}-28\sqrt{3}}{12}$

Q16: $\operatorname{sen} x = \frac{4}{5}$ $\quad \frac{\pi}{2} < x < \pi$ (2ºQ) $\operatorname{sen} x = \operatorname{sen} y$

1ºQ

$\operatorname{sen}^2 y + \cos^2 y = 1$

$\left(\frac{4}{5}\right)^2 + \cos^2 y = 1$

$\cos^2 y = 1 - \frac{16}{25}$

$\cos^2 y = \frac{25-16}{25}$

$\cos y = \sqrt{\frac{9}{25}}$

$\cos y = \frac{3}{5} \Rightarrow$ 2ºQ $\Rightarrow \cos x = -3/5$

$\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x} = \frac{4/5}{-3/5} = \frac{4}{5}\cdot\left(-\frac{5}{3}\right) = -\frac{4}{3}$

$\operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x} = \frac{-3/5}{4/5} = -\frac{3}{5}\cdot\frac{5}{4} = -\frac{3}{4}$

$\sec x = \frac{1}{\cos x} = \frac{1}{-3/5} = 1\cdot\left(-\frac{5}{3}\right) = -\frac{5}{3}$

$\operatorname{cosec} x = \frac{1}{\operatorname{sen} x} = \frac{1}{4/5} = 1\cdot\frac{5}{4} = \frac{5}{4}$

Q17: $\cos x = \frac{1}{2}$ $\quad \frac{3\pi}{2} < x < 2\pi \Rightarrow$ 4ºQ $\quad \cos x = \cos y$

$\operatorname{sen}^2 y + \cos^2 y = 1$

$\operatorname{sen}^2 y + \left(\frac{1}{2}\right)^2 = 1$

$\operatorname{sen}^2 y + \frac{1}{4} = 1$

$\operatorname{sen}^2 y = 1 - \frac{1}{4}$

$\operatorname{sen} y = \sqrt{\frac{3}{4}}$

$\operatorname{sen} y = \frac{\sqrt{3}}{2}$

4ºQ

$\operatorname{sen} x = -\operatorname{sen} y = -\frac{\sqrt{3}}{2}$

$\operatorname{tg} x = \frac{-\sqrt{3}/2}{1/2} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1} = -\sqrt{3}$

$\operatorname{cotg} x = \frac{1/2}{-\sqrt{3}/2} = \frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right) = -\frac{\sqrt{3}}{3}$

$\sec x = \frac{1}{\frac{1}{2}} = 2$ $\quad$ $\operatorname{cosec} x = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

Q.18. $\sec x = 1{,}002442 \Rightarrow \sec x = \frac{1}{\cos x} \Rightarrow \cos x = \frac{1}{\sec x}$

$\cos x = \frac{1}{1{,}002442} \approx 0{,}9976$

$\operatorname{cosec} x = \frac{1}{\operatorname{sen} x} \approx \frac{1}{0{,}06928} \approx 14{,}43$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\operatorname{sen}^2 x + (0{,}9976)^2 = 1$

$\operatorname{sen}^2 x + 0{,}9952 \approx 1$

$\operatorname{sen}^2 x \approx 1 - 0{,}952$

$\operatorname{sen} x \approx \sqrt{0{,}0048}$

$\operatorname{sen} x \approx 0{,}06928$

$\operatorname{tg} x \approx \frac{0{,}06928}{0{,}9976} \approx 0{,}06945$

$\operatorname{cotg} x \approx \frac{0{,}9976}{0{,}06928} \approx 14{,}3995$

$y = \operatorname{sen}^2 x + 2\operatorname{tg}^2 x \Rightarrow y \approx 0{,}0048 + 2 \cdot 0{,}0048$

$y \approx 0{,}0096$

Q19: $\operatorname{sen} x = \frac{1}{3}$ $\quad 0 < x < \frac{\pi}{2}$ (1º Q) $\quad \operatorname{cosec} x = \frac{1}{\operatorname{sen} x} = \frac{1}{1/3} = 3$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2 x = 1$

$\cos^2 x = 1 - \frac{1}{9}$

$\cos x = \sqrt{\frac{8}{9}}$

$\cos x = \frac{2\sqrt{2}}{3}$

$\sec x = \frac{1}{\cos x} = \frac{1}{2\sqrt{2}/3} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\operatorname{tg} x = \frac{1/3}{\frac{2\sqrt{2}}{3}} = \frac{1}{3} \cdot \frac{3}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\operatorname{cotg} x = \frac{2\sqrt{2}/3}{1/3} = \frac{2\sqrt{2}}{3} \cdot \frac{3}{1} = 2\sqrt{2}$

$y = \frac{1}{\operatorname{cosec} x + \operatorname{cot} x} + \frac{1}{\operatorname{cosec} x - \operatorname{cot} x} = \frac{1}{3 + 2\sqrt{2}} + \frac{1}{3 - 2\sqrt{2}} =$

$\frac{(3+2\sqrt{2}) + (3+2\sqrt{2})}{(3+2\sqrt{2}) \cdot (3-2\sqrt{2})} = \frac{6+4\sqrt{2}}{9 - 4 \cdot 2} = \frac{6+4\sqrt{2}}{9-8} = 6 + 4\sqrt{2}$

Q 20: $\cos x = \frac{2}{5}$ $\frac{3\pi}{2} < x < 2\pi$ (4º Q). tangente negativa
seno negativo.

$\cos x = \cos y$

1º Q

$\text{sen}^2 y + \cos^2 y = 1$

$\text{sen}^2 y + \left(\frac{2}{5}\right)^2 = 1$

$\text{sen}^2 y + \frac{4}{25} = 1$

$\text{sen}^2 y = 1 - \frac{4}{25}$

$\text{sen}^2 y = \frac{25-4}{25}$

$\text{sen} y = \sqrt{\frac{21}{25}}$

$\text{sen} y = \frac{\sqrt{21}}{5}$

$\text{sen} x = -\text{sen} y$

$\text{sen} x = -\frac{\sqrt{21}}{5}$

$tg x = \frac{-\sqrt{21}/5}{2/5} = -\frac{\sqrt{21}}{5} \cdot \frac{5}{2} = -\frac{\sqrt{21}}{2}$

$y = (1 + tg^2 x)^2 + (1 - tg^2 x)^2$

$y = 1 + 2tg^2 x + tg^4 x + 1 - 2tg^2 x + tg^4 x$

$y = 2 + 2tg^4 x$

$y = 2 + 2 \cdot \left(-\frac{\sqrt{21}}{2}\right)^4$

$y = 2 + \cancel{2}^{\div 2} \cdot \frac{\sqrt{21^4}}{\cancel{16}_{\div 2}}$

$y = 2 + \frac{21^2}{8} = 2 + \frac{441}{8} = \frac{8+441}{8}$

$y = \frac{449}{8}$

Q 21: $5 \sec x - 3 tg^2 x = 1$

$5 \cdot \frac{1}{\cos x} - 3 \cdot \frac{\text{sen}^2 x}{\cos^2 x} = 1$

$\frac{5\cos^2 x - 3 \cdot \text{sen}^2 x = \cos^2 x}{\cancel{\cos^2 x}}$

$5\cos^2 x - \cos^2 x = 3\text{sen}^2 x$

$4\cos^2 x = 3\text{sen}^2 x$

$\frac{4}{3} = \frac{\text{sen}^2 x}{\cos^2 x} = tg^2 x$

$\sec x = \frac{1}{\cos x} \Rightarrow 1 = \frac{1}{\cos x} \Rightarrow \cos x = \boxed{1}$

$tg^2 x = (tg x)^2 = \left(\frac{\text{sen} x}{\cos x}\right)^2 =$

$\frac{\text{sen}^2 x}{\cos^2 x}$

$tg^2 x = \frac{\text{sen}^2 x}{\cos x} \Rightarrow \frac{4}{3} = \frac{\text{sen}^2 x}{1}$

$\text{sen} x = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$5 \cdot \sec x - 3 \cdot tg^2 x = 1$

$5 \cdot \sec x - \cancel{3} \cdot \frac{4}{\cancel{3}} = 1$

$5 \sec x = 1 + 4$

$\sec x = \frac{5}{5} = 1$